Math 557 – Homework #1

August 24, 2000

1. Let C be the formula $(p \,\&\, \neg\, q) \Rightarrow \neg\, (p \vee r)$.
   (a) Restore all the omitted parentheses to C. (See Remark 1.1.5.)
   (b) Exhibit a formation sequence for C.
   (c) List the immediate subformulas of C, their immediate subformulas, etc., i.e., all subformulas of C.
   (d) Calculate the degrees of C and its subformulas.
   (e) Display the formation tree of C.
   (f) Write C according to various notation systems:
      i. The rules 1–3 of Definition 1.1.3:
         1. Each atom is a formula.
         2. If A is a formula then $(\neg\, A)$ is a formula.
         3. If A and B are formulas and b is a binary connective, then (AbB) is a formula.
      ii. The following alternative set of rules:
         1. Each atom is a formula.
         2. If A is a formula then $\neg\,(A)$ is a formula.
         3. If A and B are formulas and b is a binary connective, then $(A)b(B)$ is a formula.
      iii. Polish notation.
      iv. Reverse Polish notation.

2. Use truth tables to show that $((A \Rightarrow B) \Rightarrow A) \Rightarrow A$ is logically valid.

3. Use truth tables to show that $(A \,\&\, B) \Rightarrow C$ is logically equivalent to $A \Rightarrow (B \Rightarrow C)$.

4. Prove the following. (See Remarks 1.2.13 and 1.3.2.)
   (a) B is logically valid if and only if $\neg\, B$ is not satisfiable.
   (b) B is satisfiable if and only if $\neg\, B$ is not logically valid.
   (c) B is a logical consequence of $A_1, \ldots, A_n$ if and only if $(A_1 \,\&\, \cdots \,\&\, A_n) \Rightarrow B$ is logically valid.
   (d) A is logically equivalent to B if and only if $A \Leftrightarrow B$ is logically valid.