

Math 557: Mathematical Logic

Fall 2003, 3 credits, MWF 10:10–11:00 AM, 320 Sackett, Schedule 189868

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This course is suitable for all mathematics graduate students. The textbook will consist of notes provided by the instructor. A version of the notes is on line at <http://www.math.psu.edu/simpson/courses/math557/>.

1. The Propositional Calculus

Boolean operations, truth assignments, the tableau method, the Completeness Theorem, the Compactness Theorem, combinatorial applications.

2. The Predicate Calculus

Quantifiers, structures, satisfiability, tableaux, the Completeness Theorem, the Compactness Theorem.

3. Proof Systems for Propositional and Predicate Calculus

Hilbert-style systems, Gentzen-style systems, the Interpolation Theorem.

4. Extensions of the Predicate Calculus

Predicate calculus with identity, predicate calculus with operations, categoricity, countable categoricity, many-sorted predicate calculus.

5. Theories, Definability, Interpretability

Mathematical theories (groups, fields, vector spaces, graphs, ordered structures), foundational theories (arithmetic, geometry, set theory), practical completeness, definability, implicit definability, Beth's Theorem, interpretability.

6. Arithmetization and Incompleteness

Primitive recursive functions, representability, Gödel numbering, the Diagonal Lemma, Tarski's Theorem on Undefinability of Arithmetical Truth, Gödel's Incompleteness Theorem, Rosser's Incompleteness Theorem, Gödel's Theorem on Unprovability of Consistency.