

Math 557: Mathematical Logic

Homework # 2

August 28, 2000

SOLUTIONS

1. Formulate the following argument as a propositional formula.

If it has snowed, it will be poor driving. If it is poor driving, I will be late unless I start early. Indeed, it has snowed. Therefore, I must start early to avoid being late.

Solution. Use the following atoms.

s : it has snowed
 p : it is poor driving
 l : I will be late
 e : I start early

The argument can be translated as follows: $s \Rightarrow p$, $p \Rightarrow (l \vee e)$, s , therefore $(\neg l) \Rightarrow e$. Written as a single propositional formula, this becomes:

$$(*) \quad ((s \Rightarrow p) \& (p \Rightarrow (l \vee e)) \& s) \Rightarrow ((\neg l) \Rightarrow e)$$

2. Use the tableau method to demonstrate that this formula is logically valid.

Solution. The following closed tableau shows that $(*)$ is logically valid.

$$\begin{array}{c} F((s \Rightarrow p) \& (p \Rightarrow (l \vee e)) \& s) \Rightarrow ((\neg l) \Rightarrow e) \\ T(s \Rightarrow p) \& (p \Rightarrow (l \vee e)) \& s \\ F(\neg l) \Rightarrow e \\ Ts \Rightarrow p \\ T(p \Rightarrow (l \vee e)) \& s \\ Tp \Rightarrow (l \vee e) \\ Ts \\ T\neg l \\ Fe \\ Fl \\ / \quad \backslash \\ Fs \quad Tp \\ / \quad \backslash \\ Fp \quad Tl \vee e \\ / \quad \backslash \\ Tl \quad Te \end{array}$$

3. Brown, Jones, and Smith are suspected of a crime. They testify as follows:

Brown: Jones is guilty and Smith is innocent.

Jones: If Brown is guilty then so is Smith.

Smith: I'm innocent, but at least one of the others is guilty.

Let b , j , and s be the statements "Brown is innocent," "Jones is innocent," "Smith is innocent". Express the testimony of each suspect as a propositional formula. Write a truth table for the three testimonies.

Solution. The testimonies are:

$$\begin{aligned} B &: (\neg j) \& s \\ J &: (\neg b) \Rightarrow (\neg s) \\ S &: s \& ((\neg b) \vee (\neg j)) \end{aligned}$$

The truth table is:

	b	j	s	B	J	S
1	T	T	T	F	T	F
2	T	T	F	F	T	F
3	T	F	T	T	T	T
4	T	F	F	F	T	F
5	F	T	T	F	F	T
6	F	T	F	F	T	F
7	F	F	T	T	F	T
8	F	F	F	F	T	F

4. Use the above truth table to answer the following questions:

- (a) Are the three testimonies consistent?

Solution. Yes, by line 3 of the table.

- (b) The testimony of one of the suspects follows from that of another. Which from which?

Solution. The table shows that S is a logical consequence of B . In other words, Smith's testimony follows from Brown's.

- (c) Assuming everybody is innocent, who committed perjury?

Solution. If everybody is innocent, we are in line 1 of the table. Hence B and S are false, i.e., Brown and Smith lied.

- (d) Assuming all testimony is true, who is innocent and who is guilty?

Solution. If all the testimony is true, we are in line 3 of the table. Thus Brown and Smith are innocent, while Jones is guilty.

- (e) Assuming that the innocent told the truth and the guilty told lies, who is innocent and who is guilty?

Solution. Our assumption is $v(b) = v(B)$, $v(j) = v(J)$, $v(s) = v(S)$. Hence we are in line 6 of the table. Thus Jones is innocent, and Brown and Smith are guilty.