

Math 557 – Midterm Exam #2

November 4, 2005

4 problems

1. Let $M = (U_M, f_M, g_M, I_M)$ where $U_M = \{0, 1, 2, 3, 4\}$, I_M is the identity relation on U_M , and f_M, g_M are the binary operations of addition and multiplication modulo 5. Thus M is essentially just the ring of integers modulo 5. Let L be the language consisting of f, g, I . Note that M is a normal L -structure.

Write an L -sentence A such that for all normal L -structures M' , M' satisfies A if and only if M' is isomorphic to M .

2. Let G be a group. For $a \in G$ write $a^n = a \cdots a$ (n times). We say that G is *torsion-free* if for all $a \in G$, if $a \neq 1$ then $a^n \neq 1$ for all positive integers n .
 - (a) Exhibit an infinite set of sentences, S , such that for all groups G , G is torsion-free if and only if G satisfies S .
 - (b) Show that there is no finite set S as above.
3. Let A be the logically valid sentence $\exists x (Px \Rightarrow \forall y Py)$. Find a companion sequence C_1, \dots, C_n for A such that $(C_1 \wedge \cdots \wedge C_n) \Rightarrow A$ is a quasitautology.
4. Let A be the logically valid sentence $\exists x (Px \Rightarrow \forall y Py)$. Construct either a Hilbert-style proof of A or a Gentzen-style proof of A .