

Math 557 – Homework #4

October 24, 2005

1. Let $L = \{R, \dots\}$ be a language which includes a binary predicate R . Let S be a set of L -sentences. Assume that for each $n \geq 1$ there exists an L -structure (U_n, R_n, \dots) satisfying S and containing elements a_{n1}, \dots, a_{nn} such that $\langle a_{ni}, a_{nj} \rangle \in R_n$ for all i and j with $1 \leq i < j \leq n$. Prove that there exists an L -structure $(U_\infty, R_\infty, \dots)$ satisfying S and containing elements $a_{\infty i}, i = 1, 2, \dots$ such that $\langle a_{\infty i}, a_{\infty j} \rangle \in R_\infty$ for all i and j with $1 \leq i < j$.
2. Let S be an infinite set of L -sentences, and let B be an L -sentence. Prove that $S \models B$ (i.e., B is true in all L -structures satisfying S) if and only if there exists a finite set of L -sentences $A_1, \dots, A_k \in S$ such that $A_1, \dots, A_k \vdash B$ (i.e., B is provable from A_1, \dots, A_k).
3. Write out all the axioms and rules of the Hilbert-style system LH .
4. Construct a Hilbert-style proof of the sentence
$$(\exists x \forall y Rxy) \Rightarrow (\forall y \exists x Rxy).$$
5. Construct a Hilbert-style proof of the sentence
$$\neg \exists x (Sx \wedge \forall y (Eyx \Leftrightarrow (Sy \wedge \neg Eyy))).$$
6. Write out all the axioms and rules of the Gentzen-style system LG .
7. Construct a Gentzen-style proof of the sequent
$$\exists x \forall y Rxy \rightarrow \forall y \exists x Rxy.$$
8. Construct a Gentzen-style proof of the sequent
$$\rightarrow (\exists x \forall y Rxy) \Rightarrow (\forall y \exists x Rxy).$$
9. Construct a Gentzen-style proof of the sequent
$$\rightarrow \neg \exists x (Sx \wedge \forall y (Eyx \Leftrightarrow (Sy \wedge \neg Eyy))).$$