Math 557 – Homework #2

September 23, 2005

1. Assume the following predicates:

   Hx: x is a human

   Cx: x is a car

   Tx: x is a truck

   Dxy: x drives y

   Write formulas representing the obvious assumptions: no human is a car, no car is a truck, humans exist, cars exist, only humans drive, only cars and trucks are driven, etc.

2. Write formulas representing the following statements:

   (a) Everybody drives a car or a truck.

   (b) Some people drive both.

   (c) Some people don't drive either.

   (d) Nobody drives both.

3. Assume in addition the following predicate:

   Ixy: x is identical to y

   Write formulas representing the following statements:

   (a) Every car has at most one driver.

   (b) Every truck has exactly two drivers.

   (c) Everybody drives exactly one vehicle (car or truck).

4. Assume the following predicates:

   Ixy: $x = y$

   $Pxyz$: $x \cdot y = z$

   Write formulas representing the axioms for a group: axioms for equality, existence and uniqueness of products, associative law, existence of an identity element, existence of inverses.

5. Use signed tableaux to show that the following are logically valid.

   (a) $(\forall x\,(A \Rightarrow B)) \Rightarrow ((\forall x\, A) \Rightarrow (\forall x\, B))$

   (b) $(\exists x\,(A \vee B)) \Leftrightarrow ((\exists x\, A) \vee (\exists x\, B))$

   (c) $(\exists x\, A) \Leftrightarrow (\neg \forall x \neg A)$

   (d) $(\forall x\,(A \vee C)) \Leftrightarrow ((\forall x\, A) \vee C)$, provided x is not free in C

6. Let L be the language consisting of one binary predicate, R. Recall that an L-*structure*, M, consists of a nonempty set $U_M = U$, the *domain* or *universe* of M, together with a binary relation $R_M = R \subseteq U \times U$. A set of L-sentences S is said to be *satisfiable* if there exists an L-structure M such that all of the sentences of S are true in M.

   Consider the following sentences:

   (a) $\forall x\, Rxx$

   (b) $\forall x \neg Rxx$

   (c) $\forall x\, \forall y\, (Rxy \Rightarrow Ryx)$

   (d) $\forall x\, \forall y\, (Rxy \Rightarrow \neg Ryx)$

   (e) $\forall x\, \forall y\, \forall z\, ((Rxy \wedge Ryz) \Rightarrow Rxz)$

   (f) $\forall x\, \exists y\, Rxy$

   Which of subsets of this set of sentences are satisfiable? Verify your claims by exhibiting appropriate structures. Use the simplest possible structures.