

MATH 457: Mathematical Logic

Homework # 4

February 14, 2005

SOLUTIONS

Consider the following sentences:

1. $\forall x Rxx$
2. $\forall x \neg Rxx$
3. $\forall x \forall y (Rxy \Rightarrow Ryx)$
4. $\forall x \forall y (Rxy \Rightarrow \neg Ryx)$
5. $\forall x \forall y \forall z ((Rxy \wedge Ryz) \Rightarrow Rxz)$
6. $\forall x \exists y Rxy$

Which of subsets of this set of sentences are satisfiable? Verify your claims by exhibiting appropriate structures. Use the simplest possible structures.

Solution.

1,3,5,6 is satisfiable: $U_M = \{1\}$, $R_M = \{\langle 1, 1 \rangle\}$.

2,3,4,5 is satisfiable: $U_M = \{1\}$, $R_M = \{\}$.

2,3,6 is satisfiable: $U_M = \{1, 2\}$, $R_M = \{\langle 1, 2 \rangle, \langle 2, 1 \rangle\}$.

2,4,5,6 is satisfiable: $U_M = \{1, 2, 3, \dots\}$, $R_M = <$.

These are the only maximal satisfiable sets, because:

1,2 is not satisfiable.

1,4 is not satisfiable.

2,3,5,6 is not satisfiable.

3,4,6 is not satisfiable.

Note: 4,5,6 is not satisfiable in any finite domain.