

MATH 457: Mathematical Logic

Homework # 6

March 31, 2005

1. Exhibit a sentence in the predicate calculus with identity whose spectrum is $\{n \geq 1 \mid n = 3m + 2 \text{ for some } m\} = \{2, 5, 8, 11, 14, \dots\}$.
2. Let L be the language consisting of the identity predicate I and a binary predicate R . Consider the normal L -structure $M = (U_M, I_M, R_M)$ where $U_M = \{1, 2, 3, 4\}$, $I_M = \{\langle i, i \rangle \mid i = 1, 2, 3, 4\}$, and $R_M = \{ \langle i, j \rangle \mid 1 \leq i < j \leq 4 \} = \{ \langle 1, 2 \rangle, \langle 1, 3 \rangle, \dots \}$. Exhibit an L -sentence A such that, for all normal L -structures M' , M' satisfies A if and only if M' is isomorphic to M .
3. Let L be the language consisting of the identity predicate I and a 3-ary predicate P . Consider the normal L -structure $M = (U_M, I_M, P_M)$ where $U_M = \{1, 2, 3, 4\}$, $I_M = \{\langle i, i \rangle \mid 1 \leq i \leq 4\}$, and $P_M = \{ \langle i, j, k \rangle \mid i + j = k \} = \{ \langle 1, 1, 2 \rangle, \langle 1, 2, 3 \rangle, \langle 2, 1, 3 \rangle, \dots \}$. Exhibit an L -sentence A such that, for all normal L -structures M' , M' satisfies A if and only if M' is isomorphic to M .
4. Exhibit a sentence in the predicate calculus with identity whose spectrum is the set of prime numbers $\{2, 3, 5, 7, 11, \dots\}$.