

# MATH 457: Mathematical Logic

## Homework # 5

March 16, 2005

Exercises on the tableau method.

1. Consider the pair of sentences

$\forall x (Ux \Rightarrow Ax)$ : All unicorns are animals.

$\neg \exists x (Ux \wedge Ax)$ : No unicorns are animals.

- (a) Use a signed tableau to test whether this pair of sentences is satisfiable.
- (b) If it is satisfiable, use your tableau to find a structure which satisfies it.

2. Let  $A$  be the sentence

$$((\exists x Px) \Leftrightarrow (\exists x Qx)) \Rightarrow (\exists x (Px \Leftrightarrow Qx))$$

- (a) Use a signed tableau to test whether  $A$  is logically valid.
- (b) If  $A$  is not logically valid, use your tableau to find a structure in which  $A$  is false.

3. Which of the following sentences are logically valid? Which are satisfiable? Use signed tableaux to justify your answers.

- (a)  $\forall x \forall y \forall z ((Rxy \wedge Ryz) \Rightarrow Rxz)$
- (b)  $(\forall x Px) \Rightarrow (\exists x \exists y (Px \wedge Rxy))$
- (c)  $(\forall x \forall y \neg Qxxy) \Rightarrow (\forall x \neg Qxxx)$
- (d)  $(\forall x \exists y Rxy) \Rightarrow (\exists y \forall x Rxy)$
- (e)  $(\exists x \forall y Rxy) \wedge (\forall x \exists y \neg Rxy)$