

Math 457: Introduction to Mathematical Logic

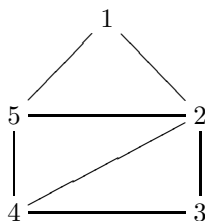
Homework # 2

Due Friday, January 28, 2005

1. Consider the following argument.

The attack will succeed only if the enemy is taken by surprise or the position is weakly defended. The enemy will not be taken by surprise unless he is overconfident. The enemy will not be overconfident if the position is weakly defended. Therefore, the attack will not succeed.

- (a) Translate the argument into propositional calculus.
 - (b) Use an unsigned tableau to determine whether the argument is logically valid.
2. Consider a graph G with vertices 1, 2, 3, 4, 5 and edges $\{1, 2\}$, $\{2, 3\}$, $\{2, 4\}$, $\{2, 5\}$, $\{3, 4\}$, $\{4, 5\}$, $\{5, 1\}$ as shown.



Let c_1, c_2, c_3 be colors. For each i and j let p_{ij} be a propositional atom saying that vertex i receives color c_j . Here the number of atoms is $5 \times 3 = 15$.

- (a) Exhibit a set of propositional formulas, $C_2(G)$, which is satisfiable if and only if there exists a coloring of G with two colors c_1, c_2 .
- (b) Show that $C_2(G)$ is not satisfiable.
- (c) Exhibit a set of propositional formulas, $C_3(G)$, which is satisfiable if and only if there exists a coloring of G with three colors c_1, c_2, c_3 .
- (d) Show that $C_3(G)$ is satisfiable.