

Math 140, Section 5

Quiz #8

March 16, 2001

SOLUTIONS

Find the asymptotes and use them to graph the functions.

1. $y = \frac{x^2 + 1}{x^2 - 2x + 1}$

Solution. By long division we have

$$y = \frac{x^2 + 1}{x^2 - 2x + 1} = 1 + \frac{2x}{x^2 - 2x + 1} = 1 + \frac{2x}{(x - 1)^2}.$$

Thus $y = 1$ is a horizontal asymptote, and $x = 1$ is a vertical asymptote. As $x \rightarrow 1$, $y \rightarrow +\infty$. When $x = 0$, $y = 1$. The graph looks like this:

2. $y = \frac{x^2 + 1}{x - 1}$

Solution. By long division we have

$$y = \frac{x^2 + 1}{x - 1} = x + 1 + \frac{2}{x - 1}$$

so $y = x + 1$ is an oblique asymptote, and $x = 1$ is a vertical asymptote. As $x \rightarrow 1^+$, $y \rightarrow +\infty$. As $x \rightarrow 1^-$, $y \rightarrow -\infty$. When $x = 0$, $y = -1$. When $x = 2$, $y = 5$. The graph looks like this: