Math 140, Section 5
Quiz #7

March 1, 2001

SOLUTIONS

$$f(x) = \begin{cases} 2x^3 + 9x^2 + 12x & \text{for } x < 0 \\ \sin x & \text{for } x \geq 0 \end{cases}$$

1. Find the critical points of $f(x)$.

   For $x < 0$ we have $f'(x) = 6x^2 + 18x + 12 = 6(x+1)(x+2)$ so there are critical points at $x = -1, -2$.

   For $x > 0$ we have $f'(x) = \cos x$ so there are critical points at $x = \pi/2, 3\pi/2, \ldots$.

   There is also a critical point at $x = 0$, because $f'(0)$ is undefined. This is because the left-hand and right-hand derivatives of $f(x)$ at $x = 0$ are unequal, namely $6x^2 + 18x + 12|_{x=0} = 12$ and $\cos x|_{x=0} = 1$.

2. Find the intervals on which $f(x)$ is increasing or decreasing. Find the local maxima and local minima of $f(x)$.

   On the interval $x < -2$, $f'(x) > 0$ so $f(x)$ is increasing. On the interval $-2 < x < -1$, $f'(x) < 0$ so $f(x)$ is decreasing. On the interval $-1 < x < 0$, $f'(x) > 0$ so $f(x)$ is increasing.

   For $x > 0$, $f(x) = \sin x$ is increasing on the intervals $0 < x < \pi/2$, $3\pi/2 < x < 5\pi/2$, etc, and decreasing on the intervals $\pi/2 < x < 3\pi/2$, $5\pi/2 < x < 7\pi/2$, etc.

   There are local maxima at $x = -2$ and at $x = \pi/2, 5\pi/2, \ldots$. There are local minima at $x = -1$ and at $x = 3\pi/2, 7\pi/2, \ldots$.

3. Find the intervals on which $f(x)$ is concave up or concave down. Find the inflection points of $f(x)$.

   For $x > 0$ we have $f''(x) = -\sin x$. Thus $f(x)$ is concave down in the intervals $0 < x < \pi$, $2\pi < x < 3\pi$, etc, and concave up in the intervals $\pi < x < 2\pi$, $3\pi < x < 4\pi$, etc. Hence there are inflection points at $x = \pi, 2\pi, 3\pi, \ldots$.

   For $x < 0$ we have $f''(x) = 12x + 18 = 6(2x+3)$. Thus there is an inflection point at $x = -3/2$. On the interval $x < -3/2$, $f''(x) < 0$ so $f(x)$ is concave down. On the interval $-3/2 < x < 0$, $f''(x) > 0$ so $f(x)$ is concave up.

   There is also an inflection point at $x = 0$.

4. Use the above information to sketch the graph of $f(x)$.

   We have $f(0) = 0$, $f(-1) = -2+9-12 = -5$, $f(-2) = -16+36-24 = -4$. The graph looks like this: