

Math 140, Section 5

Quiz #13

April 20, 2001

SOLUTIONS

Let R be the finite region in the xy -plane bounded by the curve $y = x^2$ and the line $y = 4$.

1. Find the area of R .

Solution.

The curve $y = x^2$ and the line $y = 4$ intersect at $x = \pm 2$.

$$\begin{aligned} A &= \int_a^b [f(x) - g(x)] dx = \int_{-2}^2 4 - x^2 dx \\ &= \left(4x - \frac{x^3}{3} \right) \Big|_{-2}^2 = \left(8 - \frac{8}{3} \right) - \left(-8 + \frac{8}{3} \right) = \frac{32}{3}. \end{aligned}$$

2. Find the volume of the solid of revolution obtained by rotating R about the x -axis.

Solution.

We use the washer method.

$$\begin{aligned} V &= \int_a^b \pi [f(x)^2 - g(x)^2] dx = \int_{-2}^2 \pi [4^2 - (x^2)^2] dx = \pi \int_{-2}^2 16 - x^4 dx \\ &= \pi \left(16x - \frac{x^5}{5} \right) \Big|_{-2}^2 = 2\pi \left(32 - \frac{32}{5} \right) = \frac{256}{5}\pi. \end{aligned}$$