Analysis B
Complex Analysis

Description

Complex numbers. Holomorphic functions. Cauchy's theorem. Meromorphic functions. Plane topology. Conformal maps. Other topics at time permits.

Course Objectives

The course is devoted to the exploration of complex-differentiable functions, both as a central part in pure mathematics and as a vital computational tool.

Syllabus

1. **Basics.** Geometric description of complex numbers. The complex plane and the Riemann sphere. Conformality. Linear transformations as conformal maps. Representation by complex numbers.

2. **Holomorphic functions.** Definition. Exponential and trigonometric functions. Conformality and Cauchy-Riemann equations. Relation to harmonic functions. Power series: uniform convergence, Weierstrass M-test, continuity, integrability, differentiability. Integration along curves, primitives.

3. **Cauchy's Theorem and Applications.** Goursat's theorem. Cauchy's theorem in a disk. Evaluation of integrals. Morera's theorem. Cauchy integral formulas. Cauchy estimates and Liouville's theorem. Fundamental theorem of algebra. Isolated zeros and analytic continuation. Sequences of holomorphic functions. Schwarz reflection principle.

4. **Meromorphic functions.** Zeros and poles. Laurent series. The residue formula for some domains. Jordan and "small arc" lemmas. Computation of integrals by residue calculus. Riemann's theorem on removable singularities. Essential singularities and Casorati–Weierstrass theorem. The argument principle and applications (Rouché's theorem, open mapping theorem, maximum modulus theorem). Gamma and zeta functions.

5. **Plane topology.** Simply and multiply connected domains. Jordan curve theorem (without proof). Homotopies and the winding number. General form of

Cauchy's Theorem. Roots and logarithm (including branches and cuts). Additional examples of integral evaluation using residues.

6. **Conformal maps.** Elementary conformal maps. Schwarz lemma and automorphisms of the disk and upper-half plane. Fractional linear transformations (cross ratio, behavior of lines and circles). Normal familes. Montel's theorem and the Riemann mapping Theorem.

7. **Other optional topics, as time permits.** Paley-Wiener and functions of exponential type. The prime number theorem. Entire functions and Picard's theorem. Riemann surfaces. Elliptic functions.

Other Information

Recommended text is Elias Stein, Rami Shakarchi, *Complex Analysis*, Princeton University Press 2003.