M403: Classical Analysis 1

Fall 2005, MWF 11:15-12:05, 132 E E East

Instructor. Omri Sarig, `sarig@math.psu.edu`

Prerequisite. MATH 312

Recommended textbooks. (You do NOT need to purchase these books):

(1) Kenneth R. Davidson, Allan P. Donsig: *Real Analysis with Real Applications.* Prentice-Hall. (Main text)
(2) Walter Rudin: *Principles of Mathematical Analysis.* McGraw-Hill. (Excellent, but more difficult)

Office hours. Wednesdays 7:00-8:00pm. My office is 222 McAllister.

Assessment. Grades will be assigned on the basis of 300 points:

- 100 pts for homework
- 100 pts for a midterm examination
- 100 pts for the final examination

Homework guidelines. The score for a non-submitted assignment is zero. Late assignments will not be accepted under *any* circumstance. The lowest two scores will not be taken into account. *PSU policies regarding academic integrity apply*, see `http://www.psu.edu/ufs/policies`

Syllabus

Week 1. The real line $\mathbb{R}$.

(1) Introduction, $\epsilon - \delta$ definition of the limit of a sequence, completeness.
(2) Axioms of $\mathbb{R}$, proof of completeness (from the LUB property)
(3) Countability, $|\mathbb{Q}| < |\mathbb{R}|$.

Week 2. The Euclidean space $\mathbb{R}^n$

(1) $\mathbb{R}^n$,$\|\cdot\|$, $\langle\cdot,\cdot\rangle$ and their properties (§4.1)
(2) geometric interpretation (§4.1 exercises)
(3) convergence and completeness (§4.2)

Week 3. Topology of $\mathbb{R}^n$

(1) limit points, cluster points, closed sets, intersections and unions of closed sets (§4.3)
(2) the closure and its properties
(3) open balls, open sets, open=complement of closed, intersections and unions of open sets, interior points, boundary points

Week 4. Compactness

(1) Equivalent definitions of compactness (sequential compactness, open cover property, finite-intersection property) §4.4
(2) continuation
(3) Identification of compact subsets of $\mathbb{R}^n$ (Heine-Borel)

Week 5. Continuity

(1) limits of functions $\mathbb{R}^n \to \mathbb{R}^m$, $\epsilon - \delta$ def of continuity, examples §5.1
(2) examples of discontinuity, including $f(x,y) = \begin{cases} x^2/(x^2+y^2) & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$
(3) Equivalent definitions of continuity §5.3 (sequences, open sets)

Week 6. Properties of continuous functions

(1) Arithmetic, composition, coordinate-wise continuity §5.3
(2) Extreme value theorem, via continuous image of compact is compact §5.4
(3) Uniform continuity, def and examples §5.5

Week 7. Connectedness and the intermediate value theorem

(1) Uniform continuity on compacts §5.5
(2) Pathwise connectedness, IVT 1, connectedness, IVT2 §5.6
(3) Example of a connected but not pathwise connected set (optional), Example of a space filling curve (optional)

Week 8. Normed Vector spaces

(1) definition, examples §7.1
(2) convergence, completeness, open sets §7.2
(3) all norms on $\mathbb{R}^n$ are equivalent (optional)

Week 9. Uniform convergence

(1) Limits of functions and uniform convergence §8.1
(2) **Midterm (Wednesday, October 26)**
(3) Properties of uniform convergence §8.2, 8.3

Week 10.

(1) completion of §8.3
(2) Series of functions §8.4
(3) Power series §8.5, Radius of convergence, term-by-term differentiation

Week 11. The space $C(K)$

(1) Completeness of $C(K)$
(2) Arzela-Ascoli Thm §8.6
(3) Arzela-Ascoli Thm §8.6

Week 12. Stone-Weierstrass theorem in $\mathbb{R}^n$ §10.10

Week 13. Completion of Stone–Weierstrass

(1) Completion of Stone–Weierstrass
(2) Thanksgiving
(3) Thanksgiving

Week 14. Metric spaces

(1) definition and examples of metric spaces
(2) convergence, continuity, open/closed sets
(3) completeness

Week 15. Review