MATH 231H SPECIAL ASSIGNMENT TWO

Submit on February 27th. Please prove all your assertions.

Problem 1. Read the table on page 872 of the textbook, and then do exercises 21–28 on page 874 of the textbook. Justify your choices!

Problem 2. Show that the set of points (x, y, z) satisfying the condition $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$ is a line. Describe this line in terms of the parameters a, b, c, x_0, y_0, z_0.

Problem 3. Let A, B, C be three points not on the same line, and X some other point. Write $\mathbf{a} = \overrightarrow{OA}$, $\mathbf{b} = \overrightarrow{OB}$, $\mathbf{c} = \overrightarrow{OC}$, and $\mathbf{x} = \overrightarrow{OX}$ (O is the origin).

(1) Show that X lies in the plane generated by A, B, C exactly when $(\mathbf{x} - \mathbf{a}) \cdot ((\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a})) = 0$.

(2) Find the equation of the plane passing through $A(1, 0, -1), B(0, 0, 1), C(-1, -1, 0)$.

****Problem 4.** Imagine a cylinder made of two identical rings with radius r and distance $2h$, connected by vertical pieces of rubber string. Now imagine that the top ring is rotated by an angle $0 < \theta < 180°$, but that the bottom ring is held fixed.

Find the equation for the resulting surface, and try to bring it to one of the forms given in the table at the end of section 13.6 (i.e. identify a, b, c). What happens when $\theta = 180°$?