

MATH 110
SECTION 002
MOCK QUIZ 4

1. Find a point on the y -axis that is equidistant from the points $(5, 2)$ and $(3, 10)$.

Ans: $(0, 5)$

Any point on the y -axis must have x coordinate 0. Hence we seek a point $(y, 0)$ from which the distance to $(5, 2)$ is the same as the distance to $(3, 10)$. Two applications of the distance formula yield:

$$\sqrt{5^2 + (y - 2)^2} = \sqrt{3^2 + (y - 10)^2}.$$

Squaring both sides, we have:

$$5^2 + (y - 2)^2 = 3^2 + (y - 10)^2$$

Rearranging and evaluating squares of 5 and 3, we have:

$$25 - 9 = (y - 10)^2 - (y - 2)^2;$$

$$16 = (y - 10)^2 - (y - 2)^2;$$

On the right hand side we have the difference of two squares (recall $a^2 - b^2 = (a - b)(a + b)$ and set a to be $(y - 10)$ and b to be $(y - 2)$) whence we obtain

$$16 = [(y - 10) - (y - 2)][(y - 10) + (y - 2)]$$

Simplifying terms in the square brackets, we obtain

$$16 = [-8][2y - 12]$$

It follows that

$$-2 = [2y - 12]$$

whence

$$10 = 2y$$

and so

$$5 = y$$

and the point is $(0, 5)$. NOTE THAT THIS IS NOT THE SAME AS $(5, 0)$, which lies on the x -axis but not the y -axis.

2. Find the x - and y -intercepts of the graph of the equation $y = \sqrt{3\sqrt{x} + 11}$.

Ans: x -intercept $\frac{121}{9}$, y - intercept $\sqrt{11}$

To find the y -intercept, set $x = 0$ to obtain $y = \sqrt{11}$.

To find the x -intercept, set $y = 0$ to obtain

$$0 = \sqrt{3\sqrt{x} + 11}$$

Squaring both sides, we have

$$0 = 3\sqrt{x} + 11$$

and so

$$-11 = 3\sqrt{x}$$

whence

$$\frac{-11}{3} = \sqrt{x}$$

and we square both sides again to obtain

$$\frac{121}{9} = x$$

3. Find the center and radius of the circle given by the equation $x^2 + y^2 - 4x - 12y = 24$.

Ans: center $(2, 6)$, radius 8

We 'complete the square' in both the x and y variables to obtain

$$(x - 2)^2 + (y - 6)^2 - 4 - 36 = 24$$

It follows immediately that

$$(x - 2)^2 + (y - 6)^2 = 64.$$

Comparison with the general (canonical) form of the equation for a circle shows that the center is located at $(2, 6)$ and has radius 8 (since $8^2 = 64$).

4. Determine the correct equation for the line passing through the point $(12, -11)$ and parallel to the line connecting $(-1, 5)$ and $(4, 25)$.

Ans: $y = 4x - 59$

The line connecting $(-1, 5)$ and $(4, 25)$ has slope 4 as can be verified by computing the ratio of rise $(25 - 5) = 20$ to run $(4 - (-1)) = 5$.

Hence the line through $(12, -11)$ must also have slope 4.

Given the general form of the equation of a straight line $y = mx + b$ we can impute a value for m .

Since the point $(12, -11)$ must lie on this line, it must satisfy the equation we seek to determine, and so we must have $-11 = 4(12) + b$. It follows that $b = -59$.

5. Express the statement “ y is inversely proportional to x ” as a formula. Use the information that if $x = 3$ then $y = 21$ to find the constant of proportionality.

Ans: 63

The relationship of inverse proportionality is expressed as $y = \frac{k}{x}$ where k is a constant of proportionality.

We can use the data point given to compute k , since if when $x = 3$ then $y = 21$, it follows that $21 = \frac{k}{3}$ and so it follows that $k = 63$.