

571 ANALYTIC NUMBER THEORY I, FALL 2007, PROBLEMS 8

**Return by Monday 22nd October**

1. By applying the methods of Theorem 6.9 *via* the generating function  $1/\zeta(s+1)$ , show that

$$\sum_{n \leq x} \frac{\mu(n)}{n} \ll \exp(-c\sqrt{\log x})$$

for a suitable positive constant  $c$ , and deduce that

$$\sum_{n=1}^{\infty} \frac{\mu(n)}{n} = 0.$$

2. Prove that

$$\sum_{n=1}^{\infty} \frac{\mu(n) \log n}{n} = -1.$$

Hint: The appropriate generating function is

$$-\frac{d}{ds} \left( \frac{1}{\zeta(s+1)} \right) = \frac{1}{\zeta(s+1)} \left( \frac{\zeta'}{\zeta}(s+1) \right).$$