**MATH 567 NUMBER THEORY I,**
**FALL TERM 2003, PROBLEMS 1**

*Return by Thursday 11th September*

1. Calculate $\gcd(4709, 6188)$. (If unfamiliar with Euclid's algorithm, then review Exercises 1 and 2 on page 14.)

2. Find integers $m$ and $n$ so that

$$4709m + 6188n = \gcd(4709, 6188).$$

3. Let $a$, $b$, $c \in \mathbb{Z}$. Show that the equation $ax + by = c$ has solutions in integers $x$, $y$ iff $\gcd(a, b)|c$. Assuming that the equation has a solution $x_0$, $y_0$, deduce that every solution is if the form

$$x = x_0 + t\frac{b}{\gcd(a,b)},\ y = y_0 - t\frac{a}{\gcd(a,b)} \quad (t \in \mathbb{Z}).$$

4. Suppose that $\gcd(u, v) = 1$. Show that $\gcd(u - v, u + v) = 1$ or 2.

5. Let $n_1, n_2, \ldots, n_s \in \mathbb{Z}$. Define the greatest common divisor $d$ of $n_1, n_2, \ldots, n_s$ and prove that there exist integers $m_1, m_2, \ldots, m_s$ such that $n_1m_1 + n_2m_2 + \cdots + n_sm_s = d$.

6. Discuss the solubility of $a_1x_1 + a_2x_2 + \cdots + a_sx_s = c$ in integers.

7. Find $\gcd(x^{15} - 1, x^5 - x^4 - x^3 - x^2 - x - 2)$.