

MATH 567 FALL 2008, NUMBER THEORY I, PROBLEMS 1

To be submitted by Tuesday, September 2nd

**Easier problems**

1. Given  $a|b$  and  $c|d$ , prove that  $ac|bd$ .
2. Prove that if  $n$  is odd, then  $n^2 - 1$  is divisible by 8.
3. Find the greatest common divisor  $g$  of the numbers 1819 and 3587, and then find integers  $x$  and  $y$  to satisfy

$$1819x + 3587y = g.$$

4. Let  $g > 0$  and  $b$  be given integers. Prove that the equations  $(x, y) = g$  and  $xy = b$  can be solved simultaneously if and only if  $g^2|b$ .

**Harder problems**

5. Prove that every positive integer is uniquely expressible in the form

$$2^{j_0} + 2^{j_1} + 2^{j_2} + \cdots + 2^{j_m}$$

where  $m \geq 0$  and  $0 \leq j_0 < j_1 < j_2 < \cdots < j_m$ .

6. Prove that there are no positive integers  $a, b, n$  with  $n > 1$  such that

$$(a^n - b^n)|(a^n + b^n).$$

7. Prove that any positive integer of the form  $4k + 3$  has a prime factor of the same form, and similarly for the form  $6k + 5$ . Deduce that there are infinitely many primes of the form  $4k + 3$  and similarly for  $6k + 5$