MATH 465 NUMBER THEORY, SPRING 2009, PROBLEMS 12

Return by Monday 13th April

1. Find a complete set of quadratic residues r modulo 13 in the range $1 \le r \le 12$.

2. Suppose that p is an odd prime and g is a primitive root modulo p. Prove that g is a quadratic non-residue modulo p.

3. Prove that if p is an odd prime a, $b \in \mathbb{Z}$ and $(a, p) = 1$, then

$$\sum_{n=1}^{p} \left(\frac{an+b}{p}\right)_L = 0.$$

4. Prove that if p is an odd prime, then

$$\sum_{r=1}^{p-1} \left(\frac{r(r+1)}{p}\right)_L = \sum_{s=1}^{p-1} \left(\frac{1+s}{p}\right)_L = -1.$$

Hint: Observe that for every reduced residue class r modulo p there is a unique reduced residue class s_r modulo p such that $rs_r \equiv 1 \pmod{p}$, and that for every reduced residue class s modulo p one has $s_r \equiv s \pmod{p}$ for some r.