

MATH 465 NUMBER THEORY, SPRING 2009, PROBLEMS 9

Return by Monday 23rd March

1. Solve $x^3 + 3x^2 + 2x + 3 \equiv 0 \pmod{3^j}$ for $j = 1, 2, 3, 4$.
2. Solve $x^2 + x + 7 \equiv 0 \pmod{81}$.
3. Let $f(x) = x^3 + x^2 - 5$. Show that for $j = 1, 2, 3, \dots$ there is a unique $x_j \pmod{7^j}$ such that $f(x_j) \equiv 0 \pmod{7^j}$.
4. Show that 3 is a primitive root modulo 17 and draw up a table of indices to this base modulo 17. Hence, or otherwise, find all solutions to the following congruences.
 - (i) $x^{12} \equiv 16 \pmod{17}$,
 - (ii) $x^{48} \equiv 9 \pmod{17}$,
 - (iii) $x^{20} \equiv 13 \pmod{17}$,
 - (iv) $x^{11} \equiv 9 \pmod{17}$.