MATH 465 NUMBER THEORY, SPRING 2009, PROBLEMS 7

Return by Monday 2nd March

1. Let p be a prime number. Prove that for any integers a and b, $(a+b)^p \equiv a^p + b^p \pmod{p}$.

2. Prove that if p is an odd prime and $0 < k < p$, then (assuming $0! = 1$) $(p-k)!(k-1)! \equiv (-1)^k \pmod{p}$.

3. Prove the converse of Wilson's theorem, namely that if $n > 1$ and $(n-1)! \equiv -1 \pmod{n}$, then n is prime (this is probably the world's most inefficient primality test).