

MATH 465 NUMBER THEORY, SPRING 2009, PROBLEMS 6

Return by Monday 23rd February

Congruences

1. Solve the simultaneous congruences

$$x \equiv 3 \pmod{6}$$

$$x \equiv 5 \pmod{35}$$

$$x \equiv 7 \pmod{143}$$

$$x \equiv 11 \pmod{323}$$

2. Find all solutions (if there are any) to each of the following congruences
(i) $x^2 \equiv -1 \pmod{7}$, (ii) $x^2 \equiv -1 \pmod{13}$, (iii) $x^5 + 4x \equiv 0 \pmod{5}$.
3. Eggs in basket problem (India 7c.). Find the smallest number of eggs such that when eggs are removed 2, 3, 4, 5 or 6 at a time 1 remains, but when eggs are removed 7 at a time none remain.