MATH 465 NUMBER THEORY I, SPRING 2009, PROBLEMS 5

Return by Monday 16th February

Euler's function, reduced residues

1. Suppose that $m \in \mathbb{N}$ and $(a(a-1), m) = 1$. (i) Prove that

$$1 + a + a^2 + \cdots + a^{\phi(m)-1} \equiv 0 \pmod{m}.$$

(ii) Prove that if p is a prime number with $p > 5$, then infinitely many members of the sequence 1, 11, 111, 1111, ... are divisible by p.

2. Prove that if m, $n \in \mathbb{N}$ and $(m, n) = 1$, then $m^{\phi(n)} + n^{\phi(m)} \equiv 1 \pmod{mn}$.

3. For which values of $n \in \mathbb{N}$ is $\phi(n)$ odd?

4. Suppose that m, $n \in \mathbb{N}$. Show that if every prime which divides n also divides m, then $\phi(mn) = n\phi(m)$.

5. Let m, $n \in \mathbb{N}$. (i) Prove that

$$\phi(mn) = \frac{(m,n)\phi(m)\phi(n)}{\phi\big((m,n)\big)}.$$

(ii) Prove that, if $(m, n) > 1$, then $\phi(mn) > \phi(m)\phi(n)$.