

**MATH 465 NUMBER THEORY I,
SPRING TERM 2009, PROBLEMS 4**

Return by Monday 9th February

Congruences

1. Solve where possible.
 - (i) $91x \equiv 84 \pmod{143}$
 - (ii) $91x \equiv 84 \pmod{147}$
2. Suppose that $m_1, m_2 \in \mathbb{N}$, $(m_1, m_2) = 1$, $a, b \in \mathbb{Z}$. Prove that $a \equiv b \pmod{m_1}$ and $a \equiv b \pmod{m_2}$ if and only if $a \equiv b \pmod{m_1 m_2}$.
3. Prove that when a natural number is written in the usual decimal notation, (i) it is divisible by 3 if and only if the sum of its digits is divisible by 3 and (ii) it is divisible by 9 if and only if the sum of its digits is divisible by 9.
4. Show that the last decimal digit of a perfect square cannot be 2, 3, 7 or 8.
5. Prove that, for any integer a , $6|a(a+1)(2a+1)$.