MATH 465 NUMBER THEORY, SPRING TERM 2009, PROBLEMS 3

Return by Monday 2nd February

1. Suppose that a, $b \in \mathbb{N}$. Prove that $(a,b)[a,b] = ab$.

2. Let $a \in \mathbb{N}$ and $b \in \mathbb{Z}$. Prove that the equations $(x,y) = a$ and $xy = b$ can be solved simultaneously in integers x and y if and only if $a^2 | b$.

3. Solve $11x \equiv 21 \pmod{105}$.

4. Prove that $3n^2 - 1$ can never be a perfect square.

5. Let $f(x)$ denote a polynomial of degree at least 1 with integer coefficients and positive leading coefficient.

(i) Show that if $f(x_0) = m > 0$, then $f(x) \equiv 0 \pmod{m}$ whenever $x \equiv x_0 \pmod{m}$.

(ii) Show that there are infinitely many $x \in \mathbb{N}$ such that $f(x)$ is not prime.