

**MATH 465 NUMBER THEORY,
SPRING TERM 2009, PROBLEMS 2**

GREATEST COMMON DIVISOR

Return by Monday 26th January

1. Let $a, b, c \in \mathbb{Z}$ with a and b not both zero. Prove each of the following.
 - (i) If $(a, b) = 1$ and $a|bc$, then $a|c$.
 - (ii) $\left(\frac{a}{(a,b)}, \frac{b}{(a,b)}\right) = 1$.
 - (iii) $(a, b) = (a + cb, b)$.
2. Show that if $(a, b) = 1$, then $(a - b, a + b) = 1$ or 2 . Exactly when is the value 2 ?
3. Find integers x and y such that $525x + 231y = (525, 231)$.
4. Show that if $ad - bc = \pm 1$, then $(a + b, c + d) = 1$.