

**MATH 465 NUMBER THEORY, SPRING  
TERM 2009, PRACTICE EXAM 2.**

**Note: Exam 2 will be 11:15-12:05, Wednesday 25th March 2009  
Room 116, Osmond**

1. (25 marks) Solve the simultaneous congruences

$$x \equiv 4 \pmod{19}$$

$$x \equiv 5 \pmod{31}$$

2. (25 marks) Find all solutions to the congruence  $x^{38} + 12x^{20} + 6x \equiv 0 \pmod{19}$ .
3. (25 marks) (i) Solve  $x^2 + x + 23 \equiv 0 \pmod{5}$ .  
(ii) Use the Hensel-Newton method to find all solutions to

$$x^2 + x + 23 \equiv 0 \pmod{5^2}.$$

4. (25 marks) Show that 2 is a primitive root modulo 11 and draw up a table of indices to this base modulo 11. Hence, or otherwise, find all solutions to the following congruences.

(i)  $x^6 \equiv 7 \pmod{11}$ ,

(ii)  $x^{48} \equiv 9 \pmod{11}$ ,

(iii)  $x^7 \equiv 8 \pmod{11}$ .