MATH 465 NUMBER THEORY, SPRING TERM 2009, PRACTICE EXAM 2.

Note: Exam 1 will be 11:15–12:05, Wednesday 11th February 2009
Room 116 Osmond

1. (25 marks) Suppose that $l, m, n \in \mathbb{N}$. Prove that $(lm, ln) = l(m, n)$.

2. (25 marks) (i) Show that if $(l, 6) = 1$, then $l \equiv \pm 1 \pmod 6$.
   (ii) Show that if $l \equiv m \equiv 1 \pmod 6$, then $lm \equiv 1 \pmod 6$.
   (iii) Show that if $lm \equiv -1 \pmod 6$, then either $l \equiv -1 \pmod 6$ or $m \equiv -1 \pmod 6$.
   (iv) Show that if $n \in \mathbb{N}$ and $n \equiv -1 \pmod 6$, then there is a prime number p such that $p|n$ and $p \equiv -1 \pmod 6$.
   (v) Show that there are infinitely many primes of the form $6k - 1$.

3. (25 marks) Find all pairs of integers x and y such that $922x + 2163y = 7$.

4. (25 marks) (i) Prove that if $x \in \mathbb{Z}$, then $x^2 \equiv 0$ or $1 \pmod 4$.
   (ii) Prove that $5y^2 + 2 = z^2$ has no solutions with $y, z \in \mathbb{Z}$.