MATH 421 COMPLEX ANALYSIS, FALL TERM 2004, PRACTICE EXAM 2

Note that the second exam is on Wednesday 10th November, at 9:05 in Room 109 Bouke.

1. (25 points) Show that $f'(z)$ exists at no point of $\mathbb{C}$ when (i) $f(x+iy) = x^2 - y^2 - x + i(2xy + y)$, (ii) $f(z) = e^{\overline{z}}$.

2. (25 points) Use the Cauchy-Riemann equations to show in each case that $f(z)$ and $f'(z)$ are entire, i.e. holomorphic on $\mathbb{C}$. (i) $f(x+iy) = -2xy + i(x^2 - y^2)$, (ii) $f(x+iy) = \sinh x \cos y + i \cosh x \sin y$.

3. (25 points) In each case what is the largest domain of holomorphicity of the given function?
(i) $f(z) = \frac{1}{z}$, (ii) $f(z) = \frac{1}{z^2+1}$, (iii) $f(z) = \log(z^2+1)$, where we take the branch of the logarithm with $-\pi < \Im \log w \leq \pi$.

4. (25 points) Let $\mathcal{C}$ denote the path $\mathcal{C} = \{z(t) : 0 \leq t \leq 2\}$ where $z(t) = t$ $(0 \leq t \leq 1)$, $z(t) = 1 + i(t-1)$ $(1 \leq t \leq 2)$. Evaluate

$$\text{(i)} \int_{\mathcal{C}} z^3 dz, \quad \text{(ii)} \int_{\mathcal{C}} e^z dz.$$