MATH 401 INTRODUCTION TO ANALYSIS-I, FALL TERM 2007, PROBLEMS 14

Return by Monday 10th December

1. Prove that the equation $x^{10000} + x^{100} - 1 = 0$ has a solution with $0 < x < 1$.

2. Suppose that f is continuous on $[0, 1]$ and $f(0) = f(1)$. Prove that there is an $x \in [0, \frac{1}{2}]$ such that $f(x) = f(x + \frac{1}{2})$. Hint: Consider $g(x) = f(x) - f(x + \frac{1}{2})$.

3. Prove that the line $y = 2x$ intersects the cubic curve $y = x^3 - x + 1$ in at least three distinct points.