

**MATH 401 INTRODUCTION TO ANALYSIS-I,
FALL TERM 2007, PROBLEMS 12**

Return by Monday 26th November

1. Suppose that the series

$$\sum_{n=1}^{\infty} a_n$$

converges and define the tail of the series by

$$t_m = \sum_{n=m+1}^{\infty} a_n.$$

Prove that $\lim_{m \rightarrow \infty} t_m = 0$.

2. Sketch the set of points (x, y) in \mathbb{R}^2 for which $x^4 + y^2 = 1$. Explain why

- (i) they do not define a function $f : \mathbb{R} \rightarrow \mathbb{R}$,
- (ii) they do not define a function $f : [-1, 1] \rightarrow [-1, 1]$,
- (iii) they do define a function $f : [-1, 1] \rightarrow [0, 1]$.

3. Prove that $\lim_{x \rightarrow 1} (2x - 2) = 0$.