

**MATH 401 INTRODUCTION TO ANALYSIS-I,
FALL TERM 2009, PROBLEMS 11**

Return by Monday 9th November

1. Decide the convergence of each of the following series, in each case proving your assertion.

$$\begin{aligned} & \text{(i)} \sum_{n=1}^{\infty} \frac{3}{n^3 + 2} \quad \text{(ii)} \sum_{n=1}^{\infty} \frac{4}{3n + 2} \quad \text{(iii)} \sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!} (26)^n \\ & \text{(iv)} \sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!} (28)^n \quad \text{(v)} \sum_{n=1}^{\infty} (-1)^{n-1} n^{-1/4} \quad \text{(vi)} \sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right) (-1)^n. \end{aligned}$$

2. Prove that

$$\sum_{n=1}^{\infty} x^n \frac{(n!)^2}{(2n)!}$$

converges when $|x| < 4$ and diverges when $|x| > 4$.

3. Prove that

$$\sum_{n=0}^{\infty} x^n \frac{(-1)^n}{(2n)!}$$

converges for all real x .