MATH 401 INTRODUCTION TO ANALYSIS-I, FALL TERM 2009, PROBLEMS 9

Return by Monday 26th October

1. Prove, using the definition of a limit, that

$$\lim_{n\to\infty} \frac{n}{n^2+1} = 0.$$

2. Use the definition of a limit to prove that each of the following sequences $\langle a_n \rangle$ converge.

$$\text{(i) } a_n = \frac{n^2}{cn^2+1} \text{ where } c > 0.$$

$$\text{(ii) } a_n = \frac{1+(-1)^n}{3n+2}.$$

3. Use the definition of limit to prove that the sequence $\langle (-1)^{n-1} \rangle$ diverges. Hint: Use the triangle inequality to show that for any real number l one has $2 = |a_n - l + l - a_{n+1}| \le |a_n - l| + |a_{n+1} - l|$.