

**MATH 401 INTRODUCTION TO ANALYSIS-I,
FALL TERM 2009, PROBLEMS 8**

Return by Monday 19th October

1. Prove that for all $n \in \mathbb{N}$ we have $2n + 1 < 2^{n+1}$.
2. Prove that for all $n \in \mathbb{N}$ we have $n(n + 2) \leq 2^{n+1}$.
3. Let $\mathcal{U} = \left\{ \frac{n}{n+1} : n \in \mathbb{N} \right\}$.
 - (i) Prove that \mathcal{U} is non-empty and bounded above by 1.
 - (ii) Prove that if a is a real number with $a < 1$, then there is an $n \in \mathbb{N}$ such that $a < \frac{n}{n+1}$.
 - (iii) Prove that $\sup \mathcal{U} = 1$.
4. Let $\mathcal{S} = \left\{ 2 + \frac{1}{\sqrt{n}} : n \in \mathbb{N} \right\}$.
 - (i) Prove that \mathcal{S} is non-empty and bounded below by 2.
 - (ii) Prove that if a is a real number with $a > 2$, then there is an $n \in \mathbb{N}$ such that $2 + \frac{1}{\sqrt{n}} < a$.
 - (iii) Prove that $\inf \mathcal{S} = 2$.