MATH 401 INTRODUCTION TO ANALYSIS-I,
FALL TERM 2007, PROBLEMS 8

Return by Monday 22nd October

1. Suppose that $\mathcal{A}$ is bounded above, $\mathcal{B} \subset \mathcal{A}$ and $\mathcal{B}$ is non-empty. Prove that $\sup \mathcal{B}$ exists and $\sup \mathcal{B} \leq \sup \mathcal{A}$.

2. Let $\mathcal{U} = \left\{ \frac{n}{n+1} : n \in \mathbb{N} \right\}$.
   (i) Prove that $\mathcal{U}$ is non-empty and bounded above by 1.
   (ii) Prove that if a is a real number with $a < 1$, then there is an $n \in \mathbb{N}$ such that $a < \frac{n}{n+1}$.
   (iii) Prove that $\sup \mathcal{U} = 1$.

3. Let $\mathcal{S} = \left\{ 2 + \frac{1}{\sqrt{n}} : n \in \mathbb{N} \right\}$.
   (i) Prove that $\mathcal{S}$ is non-empty and bounded below by 2.
   (ii) Prove that if a is a real number with $a > 2$, then there is an $n \in \mathbb{N}$ such that $2 + \frac{1}{\sqrt{n}} < a$.
   (iii) Prove that $\inf \mathcal{S} = 2$.