

**MATH 401 INTRODUCTION TO ANALYSIS-I,
FALL TERM 2007, PROBLEMS 7**

Return by Monday 15th October

1. Let a be any element of the open interval $(0, 1)$.
 - (i) Show that there is another $b \in (0, 1)$ with $b > a$.
 - (ii) Prove that $(0, 1)$ has no maximum.
2. Prove that for all $n \in \mathbb{N}$ we have

$$1 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}.$$

3. Prove that for all $n \in \mathbb{N}$ we have $2n + 1 < 2^{n+1}$.
4. Prove that for all $n \in \mathbb{N}$ we have $n(n + 2) \leq 2^{n+1}$.