

**MATH 401 INTRODUCTION TO ANALYSIS-I,  
FALL TERM 2009, PROBLEMS 7**

*Return by Wednesday 14th October*

1. Let  $a$  be any element of the open interval  $(0, 1)$ .
  - (i) Show that there is another  $b \in (0, 1)$  with  $b > a$ .
  - (ii) Prove that  $(0, 1)$  has no maximum.
2. Suppose that  $\mathcal{A}$  is bounded above,  $\mathcal{B} \subset \mathcal{A}$  and  $\mathcal{B}$  is non-empty. Prove that  $\sup \mathcal{B}$  exists and  $\sup \mathcal{B} \leq \sup \mathcal{A}$ .
3. Let  $\mathcal{A} = \{x : x \in \mathbb{Q}, x^3 < 2\}$ . Prove that  $\sup \mathcal{A}$  exists. Guess the value of  $\sup \mathcal{A}$ .
4. Suppose that  $x \geq -1$ .
  - (i) Prove that  $(1 + x)^2 \geq 1 + 2x$ .
  - (ii) Prove that  $(1 + x)^3 \geq 1 + 3x$ .Guess a general inequality for  $(1 + x)^n$  when  $x \geq -1$  and  $n \in \mathbb{N}$ .