

**MATH 401 INTRODUCTION TO ANALYSIS-I,
FALL TERM 2009, PROBLEMS 4**

INEQUALITIES, ROOTS

Return by Monday 21st September

1. (i) Prove that if $a < b$, then $a < \frac{1}{3}(2a + b) < b$. (ii) Prove that if $0 < a < b$, then $1/b < 1/a$.

2. Let a, b, α, β be real numbers with $b > 0, \beta > 0$ and

$$\frac{a}{b} < \frac{\alpha}{\beta}.$$

(i) Prove that $a\beta < \alpha b$.

(ii) Prove that

$$\frac{a}{b} < \frac{2a + \alpha}{2b + \beta} < \frac{\alpha}{\beta}.$$

3. Let x and y be real numbers. Prove that $x^2 - xy + y^2 \geq 0$.

4. Prove that if $0 < x$ and $0 < y$, then $x < y$ if and only if $x^3 < y^3$.

5. Simplify the following.

(i) $81^{3/4}$, (ii) $3125^{1/5}$, (iii) $8^{-4/3}$.