

**MATH 401 INTRODUCTION TO ANALYSIS-I,
FALL TERM 2007, PROBLEMS 3**

INEQUALITIES

Return by Monday 17th September

Summary of order axioms (slightly different from the textbook): There is a relation “ $<$ ” which satisfies the following axioms. a, b, c denote real numbers.

- O1. Exactly one of $a < b$, $a = b$, $b < a$ holds.
- O2. If $a < b$ and $b < c$, then $a < c$.
- O3. If $a < b$, then $a + c < b + c$ for all c .
- O4. If $a < b$ and $0 < c$, then $ac < bc$.

The expression $a > b$ means $b < a$. We also use $a \leq b$ to mean “either $a < b$ or $a = b$ ”.

1. Let x be a real number. Prove that if $0 < x$, then $0 < x^2$.
2. Let x be a real number. Prove that if $x < 0$, then $0 < -x$. Deduce that $0 < x^2$. (You may suppose that $(-x)^2 = x^2$.)
3. Let x be a real number. Prove that if $x \neq 0$, then $0 < x^2$. (You may assume the conclusions of questions 1 and 2.)
4. Let a and b be real numbers. Prove that $2ab \leq a^2 + b^2$, and if $a \neq b$, then $2ab < a^2 + b^2$.
5. Prove that for any real numbers a, b, x, y ,

$$(ax + by)^2 \leq (a^2 + b^2)(x^2 + y^2).$$