Math 529 Homework 5: Due December 5

Question 1. Let V be an oriented n-dimensional real vector bundle over the n-sphere S^n, and suppose that V is *stably trivial* (i.e., $V \oplus \epsilon^1$ is trivial). Show that the Euler class of V in $H^n(S^n) = \mathbb{Z}$ is even. (There are several ways of approaching this problem. A short one uses the Steenrod squares. However, if you don't know about Steenrod squares, try to organize the stably trivial bundles over S^n into an abelian group, and then show that this group is cyclic and generated by the tangent bundle.)

Question 2. Suppose that $\mathbb{C}P^6$ is smoothly embedded (or immersed) in a Euclidean space $\mathbb{R}^n$. Compute the Pontrjagin classes of the normal bundle to the embedding (or immersion). Deduce that n must be at least 18.

Question 3. Prove the multiplicative property of the signature (this was stated in class): if M, M' are closed oriented manifolds then $\operatorname{Sign}(M \times M') = \operatorname{Sign}(M)\operatorname{Sign}(M')$.

Question 4. Express the signature of a closed oriented 12-dimensional manifold in terms of its Pontrjagin classes. (Same method as we used in class for the 8-dimensional case.)

Question 5. Construct an exotic 11-sphere.