Math 502 Homework 7 Due Friday, March 21st

(1) (The inverse function theorem) Let U and V be open subsets of $\mathbb{C}$ and let $f\colon U \to V$ be a holomorphic bijection.

Show that f is a homeomorphism (that is, show that the inverse function f^{-1} is continuous; use the open mapping theorem). Show further that f is a biholomorphic equivalence (that is, show that f^{-1} is differentiable).

(2) Construct a biholomorphic equivalence between the set

$$\{z \in \mathbb{C} : |z \pm 1| < \sqrt{2}\}$$

and the unit disc.

(3) Let X be a compact metric space. Prove the converse to Ascoli's theorem, that is, a compact subset of $C(X)$ is equicontinuous.