

Math 502 Homework 6 Due Friday, February 29th

(1) The function $f(z)$ is entire, and it is known that $|f(z)| \leq 1$ whenever $|z| = 1$ and that $|f(z)| \leq 10$ whenever $|z| = 10$. Show that $|f(z)| \leq 5$ whenever $|z| = 5$. What can be said about $|f(0)|$?

(2) Let \mathbb{U} denote the open unit disk. Suppose that $f: \mathbb{U} \rightarrow \mathbb{U}$ is a holomorphic map with two distinct fixed points. Prove that f is the identity map.

(3) Does there exist a surjective holomorphic map from the unit disk to \mathbb{C} ? Does there exist a surjective holomorphic map from the unit disk to the Riemann sphere?