

Math 502 Homework 3 Due Friday, February 8th

(1) Evaluate the integral

$$\oint \frac{\cos z}{z} dz$$

around the unit circle. Deduce that

$$\int_0^{2\pi} \cos(\cos \theta) \cosh(\sin \theta) d\theta = 2\pi.$$

(2) Suppose that the function f has a Taylor expansion

$$f(z) = \sum_{n=0}^{\infty} c_n z^n$$

valid in \mathbb{C} . Prove that for each $R \geq 0$,

$$\sum_{n=0}^{\infty} |c_n| R^n \leq 2M(2R),$$

where $M(r) = \sup\{|f(z)| : |z| = r\}$.

(3) Let f be a non-constant holomorphic function defined on an open set Ω that contains the closed unit disc. Suppose that f maps the unit circle to itself (i.e., $|z| = 1$ implies $|f(z)| = 1$). Show that f maps the open unit disc *onto* itself.