Math 501 Homework 8 Due Monday, November 10th

(1) Let Ω_1 and Ω_2 be connected, open, bounded subsets of $\mathbb{C}$. Suppose that there is a biholomorphic equivalence $\varphi\colon \Omega_1 \to \Omega_2$ which extends to a homeomorphism $\overline{\Omega_1} \to \overline{\Omega_2}$.

Show that if $u_2\colon \Omega_2 \to \mathbb{R}$ is a solution to the Dirichlet problem on Ω_2 with boundary data g, then $u_1 = u_2 \circ \varphi\colon \Omega_1 \to \mathbb{R}$ is a solution to the Dirichlet problem on Ω_1 with boundary data $g \circ \varphi$.

(2) In the situation of question 1, show that the quantity

$$\iint_{\Omega_j} \left(\frac{\partial u_j}{\partial x}\right)^2 + \left(\frac{\partial u_j}{\partial y}\right)^2 \, dxdy$$

is the same whether $j = 1, 2$. Compute this quantity when Ω is an annulus of inner radius a and outer radius b, and $g = 0$ on the inner boundary, $g = 1$ on the outer boundary.

Deduce that two such annuli are biholomorphically equivalent if and only if the ratio b/a is the same for each of them. (You may assume that a biholomorohic equivalence extends to a homeomorphism of the closed annuli.)

(3) Let f be a nonconstant elliptic function. It was shown in class that f assumes each value the same number of times in any period parallelogram.

Show that this number is at least 2. (Hint: Think how often f assumes the value ∞. What does the Residue Theorem tell you about the residue(s) at the poles(s) of f?)