Math 501 Homework 2 Due Friday, September 19th

(1) Two non-intersecting circles $\mathcal{C}$ and $\mathcal{C}'$ are given in the complex plane. Show that one can find a Möbius transformation that maps them to two *concentric* circles.

A *circle chain* between $\mathcal{C}$ and $\mathcal{C}'$ is a sequence of circles $\mathcal{C}_1$, $\mathcal{C}_2$, and so on, which lie between $\mathcal{C}$ and $\mathcal{C}'$ and each of which is tangent to $\mathcal{C}$, to $\mathcal{C}'$, and (if $n > 1$) to the previous circle in the chain. The circle chain is *closed* if $\mathcal{C}_n = \mathcal{C}_1$ for some n. Show that, if one circle chain between $\mathcal{C}$ and $\mathcal{C}'$ is closed, then every such circle chain is closed (*Steiner's porism*).

(2) True or false: There exists a sequence of complex polynomials $p_n(z)$ such that $p_n(z) \to 1/z$ uniformly on the unit circle $\{z : |z| = 1\}$? Give careful reasons.

(3) By considering the contour integral

$$\oint \frac{dz}{z}$$

taken around an elliptical contour whose Cartesian equation is $x^2/a^2 + y^2/b^2 = 1$, show that

$$\int_0^{2\pi} \frac{dt}{a^2 \cos^2 t + b^2 \sin^2 t} = \frac{2\pi}{ab}.$$