

**Math 497C Homework 9 — Due December 1st**

- (1) Show that

$$\Gamma_{kh}^k = \frac{\partial \log \sqrt{-g}}{\partial x^h}$$

where  $g$  is the determinant of the matrix  $g_{ij}$ . (Hint: To differentiate the determinant consider its expansion in terms of minors and cofactors.)

- (2) Show that the Ricci tensor can be written in the form

$$R_{ik} = \frac{\partial \Gamma_{ik}^h}{\partial x^h} - \frac{\partial^2 \log \sqrt{-g}}{\partial x^i \partial x^k} + \Gamma_{ik}^p \frac{\partial \log \sqrt{-g}}{\partial x^p} - \Gamma_{ih}^p \Gamma_{pk}^h.$$

- (3) The *scalar curvature* is the contraction of the Ricci tensor,  $R = g^{ij} R_{ij}$ . Compute the scalar curvature of an  $n$ -dimensional unit sphere in  $(n+1)$ -dimensional euclidean space.