

Math 497C Homework 6 — Due November 7th

(1) Verify the assertion, made in class, that if a patch of surface has metric tensor $g_{11} = 1$, $g_{12} = 0$, and g_{22} a function of x_1 only, then the Gaussian curvature is given by

$$K = \frac{-1}{2\sqrt{g_{22}}} \frac{\partial}{\partial x_1} \left(\frac{1}{\sqrt{g_{22}}} \frac{\partial g_{22}}{\partial x_1} \right).$$

(2) Let V^i be a vector field on spacetime, and let V_i (with index lowered) be the corresponding covector field. Show that V^i is a Killing field if and only if

$$V_{i;j} + V_{j;i} = 0$$

for all i, j .

(3) If V^i and W^j are two vector fields their *commutator* is

$$X^j = V^i W^j_{,i} - W^i V^j_{,i}.$$

Verify that X^j is a vector field (ie twiddles properly). Show also that the commutator of two Killing fields is again a Killing field.