

Math 497C Homework 6 — Due October 31st

(1) A pole vaulter carrying a 20ft pole PQ runs with speed $\sqrt{3}c/2$ into a 10ft room. Show that, as measured in the frame of the room, the pole is 10ft long.

As the end Q reaches the far wall, an umpire standing by the door of the room closes it. Explain with the aid of a space-time diagram how this is possible when, in the vaulter's coordinate system, the pole has length 20ft but the room is only 5ft long.

If the vaulter holds the end P , is he (i) outside the room (ii) inside the room or (iii) at the door when he first feels the shock of the end Q striking the wall?

(2) Let X and Y be future-pointing timelike four-vectors, and let $Z = X + Y$. Show that

$$\sqrt{g(Z, Z)} \geq \sqrt{g(X, X)} + \sqrt{g(Y, Y)}.$$

When does equality hold? What is the analogous statement in Euclidean geometry?

(3) A particle of rest mass m moving with speed u collides with another such particle which is at rest. After the collision the particles have speeds v and w and the angle between their paths is θ (all measurements are taken in a fixed inertial frame). Show that

$$\cos \theta = \frac{c^2}{vw} \left(1 - \sqrt{1 - \frac{v^2}{c^2}} \right) \left(1 - \sqrt{1 - \frac{w^2}{c^2}} \right).$$

Show also that this formula yields the Newtonian value $\theta = \pi/2$ when v and w are small compared with c .

(Bubble chamber photographs of high-energy collisions show many events with θ much smaller than $\pi/2$; this is evidence for special relativity.)