

### Math 497C Homework 3 — Due September 26th

(1) The *catenary* is the plane curve with equation  $y = \cosh x$ . The *catenoid* is the surface of revolution obtained by rotating the catenary about the  $x$ -axis. Give a parameterization of the catenoid and find its metric tensor (a.k.a. first fundamental form) in your parameterization.

(2) The *helicoid* is the surface parameterized by

$$\mathbf{r}(u, v) = (u, v \cos u, v \sin u).$$

Sketch this surface and find its metric tensor. Show further that the catenoid and the helicoid are locally isometric.

(3) The *tractrix* is the plane curve given by the parametric equation

$$x = \log \cot(u/2) - \cos u, \quad y = \sin u, \quad 0 < u < \pi/2.$$

The *tractoid* is the surface obtained by revolving the tractrix about the  $x$ -axis. Prove that the total area of the tractoid is  $2\pi$ .