Math 497C Homework 2 — Due September 19th

(1) A group G acts on a space X.
The *stabilizer* G_x of x is the set of all $g \in G$ that fix x, that is,

$$G_x = \{g \in G : gx = x\}.$$

Show that the stabilizer of x is a subgroup of G. (This means that the product of two elements of G_x is an element of G_x, and the inverse of an element of G_x is an element of G_x.)

(2) (Continuation) Two subgroups H and H' of G are *conjugate* if there is $g \in G$ such that $h \in H$ if and only if $g^{-1}hg \in H'$.

The action of G is *transitive* if, for any two points $x, x' \in X$, there is a group element g such that $gx = x'$. Show that if the action is transitive then all the stabilizers of points of X are conjugate.

Is the converse true? — that is, if all the stabilizers are conjugate, must the action be transitive?

(3) Two particles of masses m_1 and m_2, and velocity vectors $\mathbf{u}_1$ and $\mathbf{u}_2$, undergo an elastic collision; as a result of the collision their velocity vectors are changed to $\mathbf{v}_1$ and $\mathbf{v}_2$. The *law of conservation of momentum* says that

$$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v_2};$$

and the *law of conservation of energy* says that

$$m_1|\mathbf{u}_1|^2 + m_2|\mathbf{u}_2|^2 = m_1|\mathbf{v}_1|^2 + m_2|\mathbf{v_2}|^2.$$

Show that, if these laws hold in a given frame of reference, then they also hold in any other frame of reference related to the given one by a Galilean transformation.

(4) Investigate the invariance of the *D'Alembertian* operator

$$c^2\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}$$

under Galilean transformations.