Math 411 Homework 3 Due Thursday, February 17th

1. Problem 3.7.4 in Strogatz (page 90). This extends and makes more realistic our simple model of fish harvesting from the previous homework set. Since this is a long problem I will let it count for two regular homework problems (i.e. 10 points). Hints: In part (b) the 'dimensionless quantities' are just rescaled versions of N, t, A and H respectively. For instance, one can start by taking $x = (1/K)N$. In parts (d) and (e) the bifurcation parameter is h; a is considered to be a constant. See page 71 for the explanation of the term 'stability diagram' used in (f).

2. Recall that the normal form for a transcritical bifurcation is $\dot{x} = rx - x^2$, where r is a parameter.

Suppose that we introduce a small 'imperfection' a, so that the equation becomes

$$\dot{x} = a + rx - x^2.$$

On the same axes of r and x sketch bifurcation diagrams for this equation when $a > 0$, when $a = 0$, and when $a < 0$. What kinds of bifurcations occur in the three cases? Justify your answer.

3. Consider the nonlinear oscillator equation

$$\dot{\theta} = \omega + a \sin\theta$$

that was discussed in class. Thinking of ω as fixed and a as a parameter, where do bifurcations occur?

Let $x = \cos\theta$, $y = \sin\theta$. Derive the differential equations

$$\dot{x} = -y(\omega + ay), \quad \dot{y} = x(\omega + ay)$$

for x and y. Use PPLANE to plot the solutions to this system for $\omega = 1$, $a = 0.5$, and various initial conditions. Describe your results and explain how they relate to your answer to the first part of the question.

4. Consider the system of differential equations

$$\dot{x} = y - x, \quad \dot{y} = 3y - 4x.$$

Solve these equations analytically. (You will find it helpful to put $z = y - 2x$ and look at a differential equation for z.)

Now use PPLANE to plot some solution curves, and discuss how these results are compatible with your analytic solution.